

# **Savitribai Phule Pune University**

(Formerly University of Pune)

**Two Year Degree Program in Statistics** 

(Faculty of Science & Technology)

Revised Syllabi for

# M.A./M.Sc. (Statistics) Part-I

(For Colleges Affiliated to Savitribai Phule Pune University)

Choice Based Credit System Syllabus To be implemented from Academic Year 2019-2020

#### Title of the Course: M. A. / M. Sc. (Statistics)

#### **Preamble:**

M. A. / M. Sc. Statistics program is of minimum 80 credits spread over four semesters. This program is offered at the colleges affiliated to the Savitribai Phule Pune University. The program emphasizes both theory and applications of statistics and is structured to provide knowledge and skills in depth necessary for the employability of students in industry, other organizations, as well as in academics. Accordingly, the program has important features such as individual/ group projects, elective courses and courses on standard software packages such as MATLAB, MINITAB, SYSTAT, SPSS , R. Syllabus of the first two semesters covers core courses. The second year syllabus contains both core, elective and open courses. It is possible for the students to study basic courses from other disciplines such as economics, life sciences, computer science, mathematics in place of electives.

Introduction:

(a) M. A./ M. Sc. Statistics program will be conducted under credit system in four semesters. There will be 20 credits in each semester for a total of 80 credits. One credit is taken to be equivalent to 15 clock hours of teaching.

(b) The program consists of core courses which may be compulsory or electives.

(c) In addition, there are lab courses (practical) and a project course.

(d)Some courses are termed Open Courses (O). The open courses are those which can be offered to other departments.

(e)For every course, there will be Continuous Internal Assessment (CIA)

conducted by department or college and End of Semester Examination (ESE) conducted by the University at the end of semester.

# Structure of the Syllabus:

# Following is the structure of two year M.A. / M.Sc. Statistics program.T: TheoryP: PracticalO: OpenC: CompulsoryE: Elective

Course	T/ P	<b>O/C/ E</b>	Title	Number	ESE	Marks	
Code				of credits	Duration	Assigned	
Semester I							
ST - 11	Т	C	Basics of Real Analysis and Calculus	4	3 Hours	100	
			Calculus				
ST - 12	Т	С	Linear algebra and	4	3 Hours	100	
			Numerical methods				
ST - 13	Т	С	Probability Distribution	4	3 Hours	100	
ST - 14	Т	С	Sampling Theory	4	3 Hours	100	
ST - 15	P	С	Practical I	4	3 Hours	100	
			Total	20		500	
Semester II							
ST - 21	Т	С	Probability Theory	4	3 Hours	100	
ST - 22	Т	С	Regression Analysis	4	3 Hours	100	
ST - 23	Т	C	Statistical Inference I	4	3 Hours	100	
ST - 24	Т	C	Multivariate Analysis	4	3 Hours	100	
ST - 25	Р	С	Practical II	4	3 Hours	100	
			Total	20		500	

# Semester I

## ST11 : Basics of Real Analysis and Calculus

#### Unit 1

Review of Set theory, Set of real numbers, Supremum and infimum of sets of real numbers, real field, odered set and field, Archimedean principal, countable and uncountable sets, countability of rational numbers, uncountability of real numbers, Metric space, Properties of metric space Neighborhood points, Exterior and interior points, boundary points, limit points, open, closed and compact sets. Bolzano – Weierstrass and heine- Borel theorem. (10L)

## Unit 2

(i) Sequence of real numbers, limit of sequence and its properties, Convergence and divergence of sequence Cauchy sequence and related theorems (Cauchy criteria of convergence), subsequence and their convergence and divergence, convergence of bounded monotone sequence.

(ii)Series of real numbers, convergence and divergence of series of real numbers, test for convergence (root test, ratio test), absolute convergence (without proof), uniform convergence, power series, radius of convergence of power series (Binomial, Exponential, geometric and log series), term by term differentiation (integration) of absolute convergent series, change of order of summation of series. (15L)

## Unit 3

(i)Revision: derivative of function of single variable, Mean value theorem, Taylor's series expansion

(ii) Multivariate calculus: explicit and implicit functions, continuity, partial derivatives, differentiability, partial derivatives of higher order, Hessian matrix, Taylor's theorem, extreme values, differentiation with respect to vector and matrix, jacobian of transformation. (15L)

## Unit 4

(A)Riemann and Riemann – Stieltjes integral: Partition of interval, norm of partition, finer partion, tagged partion, Upper and Lower Riemann and R-S sums, order relation between Upper and Lower Riemann sums. Effect of finer partition on the difference between Upper and Lower Riemann sums. Necessary and sufficient condition for a function to be R and R-S integrable Properties with proof (i) Continuous bounded function is R and R-S integrable (ii) Monotonic bounded function is R and R-S integrable(iii) Fundamental theorem of integral calculus.

**(B) Improper integrals:** Definition, convergence of an integral, P- integral, exponential integral test for convergence (comparison test), convergence of beta and gamma integrals, relationship between beta and gamma functions

(C) **Double integrals**: Definition, properties, change of order, iterated integrals, Fubini's theorem, differentiation under integral sign (Leibnitz rule) and transformation of variables. (20L)

#### **Books Recommended:**

- 1. Ajitkumar and S. Kumaresan (2014), A Basic Course in Real Analysis (Chapman and Hall)
- 2. Rudin W. (1985), Principles of Mathematical Analysis (McGraw-Hill)
- 3. Apostol T. M. (1975) Mathematical Analysis: A Modern Approach to Advanced Calculus (Addison Wesley)
- 4. Bartle R. G. (1976), Elements of Real Analysis (Wiley)
- 5. Malik S. C. & Arora S. (1991), Mathematical Analysis (Wiley Eastern Limited 2nd edition)

6.Goldberg R. R. (1964), Methods of Real Analysis- Blasdell Publishing company, New York, U.S.A.

- 7. Bartle G.R. & Sherbert D. R. (2000), Introduction to Real Analysis- John Wiley & Son Inc.
- 8. Royden (1988), Principles of Real Analysis (Macmillian)

9. S R Ghorpade and B V Limaye(2000), A Course in Calculus and Real Analysis, Springer

10. 9. S R Ghorpade and B V Limaye(2009), A Course in Multivariate Calculus and Analysis, Springer

## Semester I

## ST 12: Linear algebra and Numerical methods

#### Unit 1

Vector Space, Subspace, Linear dependence and independence, Basis of a vector space, dimension of vector space, orthogonal and orthonormal basis, Gram –Schmidt orthogonalization, Matrix algebra, special types of matrices, rank, inverse and determinant of a matrix and their properties, Orthogonal and idempotent matrix and their properties, Projection theorem, linear transformation, linear equationsSolution space and null Space. (15 L)

## Unit 2

Generalized (g) inverse and Moore-Penrose g- inverse (MP g-inverse), its properties and Examples, system of homogeneous and non- homogeneous linear equations, solution space and null Space. (12L)

Statistics

## Unit 3

Characteristic roots (eigen values) of real matrices, right and left characteristics vectors (eigen vectors), Independence of characteristics vectors corresponding to distinct Characteristic roots, Algebraic and geometric multiplicity, Spectral decomposition, power of a matrix, Cayley-Hamilton theorem, singular value decomposition

(15 L)

## Unit 4

Introduction of quadratic forms, reduction and classification of a quadratic form, simultaneous reduction of two quadratic forms, Maxima and minima of a quadratic form, Properties of a quadratic form for orthogonal and idempotent matrices. (12 L)

## Unit 5

Newton Raphson iterative method for two or more simultaneous transcendental equations, Newton's bivariate interpolation formula, Simson's, Trapezoidal rule for bivariate integrals

(6L)

#### **Books Recommended:**

1. Bapat, R.B. (2011). Linear Algebra and Linear Models. Springer and Hindustan Book Agency.

2. Beezer, R. A. (2004). A First Course in Linear Algebra, Congruent Press, Washigton

3. Hohn, F. E. (1973). Elements of Matrix Algebra, Macmillan

4. Kollo, T. and Rosen, D. von (2005). Advanced Multivariate Statistics withMatrices, Springer, New York.

5. Kumaresan, S. (2000). Linear Algebra: A Geometric Approach, Prentice Hall

6. Lay, D. C. Lay, S. R. and Mc Donald, J. J. (2016) .Linear Algebra and Its Applications, Fifth Edition, Pearson, Boston.

7. RamachandraRao, A. and Bhimasankaram, P. (2000). Linear Algebra. Hindustan Book Agency

8. Rao, C. R. (1995). Linear Statistical Inference and Its Applications, Wiley

9. Searle, S. R. (1982). Matrix Algebra Useful for Statistics, John Wiley, New York

10. Sastri (2009, fourth edition) Prentice Hall : Introductory methods of Numerical Analysis.

11. Rajaraman (1993): Prentice Hall, Computer oriented Numerical Methods.

(3L)

# Semester I

## **ST 13: Probability Distributions**

#### Unit 1

(i) **Random Variable:** Brief introduction to sigma field, probability space, set function, measure, finite measure, probability measure, axioms of probability, Borel field, Borel measurable function, random variable as a measurable function on probability space. (3L) (ii) **Distribution function:** CDF of random variable, continuity theorem of limit of probability (without proof), characteristic properties of CDF (with proof). Decomposition of CDF, mixture of distribution function is CDF (ii) If F(.) is a CDF then  $F^n(.)$ , 1-(1-F(.))<sup>n</sup> are CDF for n positive integer. (iv) Convex combination of CDFs is CDF. Bivariate random variable. CDF of bivariate random variable and its characteristic properties. Identification of given function whether it is CDF. (6L)

(iii) Symmetry: Symmetric probability distribution around a, Concept, mean median mode need not coincide. Results: (i) Odd ordered central moments of symmetric distribution are zero. (ii) Sum and difference of random variables with symmetric distribution is symmetric. (3L) (iv) Transformations: Transformation of random variable of the type (i) one to one on to (ii) monotonic (iii) non monotonic. If g(.) is a nonlinear function of rv X with mean  $\mu$  and varience  $\sigma^2$ then  $E[g(X)] \approx g(\mu) + g'(\mu) \sigma^2$ ,  $Var(g(X) \approx (g'(\mu))^2 \sigma^2$ 

#### Unit2

(i)Expectation of random variable, necessity of existence of absolute moments, non uniqueness of moments, existence of r <sup>th</sup>order moments if s <sup>th</sup>order moment exists (r  $\leq$  s). (2L) (ii)MGF: Existence of MGF, properties, MGF of symmetric distributions. (1L) (iii)PGF: Properties, moments using PGF, probability distributions of X+Y, X-Y when X and Y are not identically distributed random variables. Compound distribution and its PGF. Wald's identity E(X<sub>1</sub>+X<sub>2</sub>+ - - + X<sub>N</sub>) = E(N)E(X<sub>i</sub>) where X<sub>i</sub>'s are iidrvs. and independent of N. Var (X<sub>1</sub>+X<sub>2</sub>+ - - + X<sub>N</sub>).Results (i) If P<sub>1</sub>(s) and P<sub>2</sub>(s) are PGF of independent rv s then P<sub>1</sub>(s) x P<sub>2</sub>(s) is a PGF (ii) P<sub>1</sub>(P<sub>2</sub>(s)) is PGF(iii)[P<sub>1</sub>(s)]<sup>n</sup> is a PGF n positive integer.(iv) Px(s)= P<sub>-x</sub>(s) if X is symmetric around zero. Convolutions of random variables .Distributions of X+Y, X-Y incase of U(0, 1), Normal, exponential etc.

(iv)Characteristic function:characteristic function and properties, conjugate pairs of distributions, Parseval relation, uniqueness theorem (6L)

(v)Random vector:Joint and marginal distributions, mixed moments, covariance matrix, Condtional mean and variance, Results (i)EE(X|Y)=E(X). (ii)Var(X)=EVar(X|Y)+VarE(X|Y). (iii)If E(Y|x)=a +bx then b is the regression coefficient of y on x. MGF of random vector. (vi)Regression:Conditional expectation as regression function (6L)

### Unit 3

## (i)Bivariate Poisson Distribution:

Definition, derivation of pmf, mgf of bivariate Poisson distribution, marginal pmf of variables, Cgf, correlation coefficient, conditional distribution of X1|X2=x2, problems. (3L)

## (ii)Bivariate exponential distribution:

Types of bivariate exponential distribution:

- 1) Marshall –Olkin model: joint distribution, marginal distribution, lack of memory property, Properties of Marshall –Olkin model
- 2) Fruend bivariate exponential model
- 3) Moran's bivariate exponential model
- 4) Gumblel's bivariate exponential model

#### Theorem:

Decomposition of bivariate exponential distribution into absolutely continuous and singular parts. (6L)

## (iii)Dirichlet Distribution:

Definition, derivation of joint pdf ,particular case of dirichlet distribution, Marginal distribution of X and

Y (for particular case k=3) .Conditional distribution of X|Y and Y|X. (3L) Unit 4

## (i)Review of order Statistics:

Definition, Marginal pdf of  $X_{(r)}$ , Jointpdf of  $(X_{(r)}, X_{(s)})$ , PDf of sample range ,corr $((X_{(r)}, X_{(s)})$  when random sample from U(0,1), problems based on exponential . Distributions of spacings and standardized spacings

(ii)Probability integral transformation. Quantile function.

(iii)Distribution free statistics: Sign test, Wilcoxon sign rank test ,kolmogorov –smirnov test, Run test.

(iv)Quadratic forms,: Quadratic forms, Classification of quadratic forms Sampling distribution of quadratic forms and linear forms for random sample from normal distribution.distribution of quadratic forms.All result require for Fisher Cochran theorem, Fisher Cochran theorem.

(v)Non-central distribution:Non-central chi square distribution, derivation of pdf, mgf, mean variance, applications.Non-central t-distribution distribution, derivation of pdf, mgf, mean variance, applications.

Non-central F- distribution, derivation of pdf, mgf, mean variance, applications.

## **Books Recommended**

1. Berger, R. and Casella G. (2002). Statistical Inference, Duxbury Resource Center, Second Edition.

2. Dasgupta, A. (2010) Fundamentals of Probability: A First Course, Springer, NewYork.

3. Hogg, R. V., McKean, J. W. and Craig, T. T. (2005). Introduction to Mathematical Statistics, Sixth Edition, Pearson Prentice Hall, New Jersey.

4. Rao, C. R. (2002). Linear Statistical Inference and Its Applications, Wiley

5. Rohatgi, V. K. & A. K. M. E Saleh (2001). Introduction to Probability and Statistics, Wiley, New York.

# Semester I

# **ST 14: Sampling Theory**

## Unit 1

Review of finite population sampling techniques (random and non-random), SRSWR and SRSWOR: Inclusion probabilities, related results on estimation of population total, Determination of sample size for pre-specified variance, pre-specified error in the estimation, pre-specified width of the confidence interval, pre-specified relative error in the estimation, Simple random sampling for the proportion, Estimation of proportion for the more than two classes, Inverse Sampling (Sampling for the rare attribute) and estimator of the population mean and its variance. Probability Proportional to Size with Replacement (PPSWR) methods, cumulative total method and Lahiri's method for estimation problem, estimation of finite population mean andtotal.

Sampling with varying probability without replacement, Ordered and Unordered estimates, Horwitz–Thompson estimator, its variance and properties, midzuno scheme of sampling, Yates-Grundy sampling estimate, Murthy's estimate. [15L]

## Unit 2

Stratified sampling, comparison of allocation problem of allocation in stratified sampling, construction of strata, deep stratification, The method of Collapsed strata, post stratification, estimator of population mean and variance of estimator of population mean under post stratification, Stratified random sampling for the proportion, comparison of stratified random sampling with simple random sampling.

Use of supplementary information for estimation, ratio estimator of population mean, its bias and mean square error, unbiased ratio type estimators of population mean, variance of estimator of population mean underit, , ratio estimator for the stratified random Sampling, regression method of estimation, estimator of population mean, its bias and mean square error of the Estimator, comparison of estimator of population mean under ratio, regression and simple random sampling,

Jack-Knife and bootstrap method of estimation, estimate of bias and standard error

[15L]

## Unit 3

Systematic sampling, sample mean and its variance, Comparison of systematic sampling under linear trend, Yates corrected estimator, Centered systematic sampling, Balanced systematic sampling and Modified systematic sampling, circular systematic sampling, two dimensional systematic sampling (Aligned and Unaligned Systematic sampling), comparison of systematic sampling with random sampling and stratified sampling, PPS systematic sampling.

Cluster sampling with clusters of equal sizes, estimation of population mean and its standard error, Relative efficiency of cluster sampling w.r.t. SRSWOR, Effect of cluster size on relative efficiency, unbiased estimator of relative efficiency, cluster sampling as a one way ANOVA, Optimum value of the cluster size, cluster sampling for the proportion, Cluster sampling with cluster of unequal

unequal sizes ,bias in estimator of population mean , bias in the estimator and its MSE, unbiased estimator and relative efficiency of unequal cluster sampling, PPS cluster sampling, estimation of population mean. [15L]

## Unit 4

Two stage sampling with equal and unequal second stage units, estimation of population mean and its standard error, optimum value of the number of clusters and second stage units in the two stage sampling having equal second stage units.

Two phase sampling, ratio and regression estimator of population mean under two phase sampling, bias in the estimator and its MSE, Sampling and non– sampling errors, response errors, mathematical model for Response errors, Hansen Hurwitz technique, Randomized Response Technique (RRT), Warner's randomized response technique. [15L]

Books Recommended

- 1. Arnab, R. (2017). Survey Sampling: Theory & Applications, Academic Press
- 2. Chaudhuri, A. (2014). Modern Survey Sampling, CRC Press
- 3. Cochran, W.G. (1984). Sampling Techniques, Wiley.
- 4. Des Raj and Chandhok, P. (1998). Sample Survey Theory, Narosa.
- 5. Murthy M.N.(1977) Sampling theory and methods (Statistical PublishingSociety)

(1)

- 6. S. Sampath (2005) Sampling theory and methods (Narosa)
- 7. Singh, D. and Chaudhary F.S (1986). Theory and Analysis of Sample Survey Designs, Wiley Eastern Limited.
- 8. Singh, S. (2003). Advance Sampling Theory and Applications (Volume I and II), Kluwer Academic Publishers.
- 9. Sukhatme, P.V, Suktatme, B.V.,Sukhatme, S. and Asok, C. (1984).Sampling Theory of Surveys with Applications, Indian Society for Agricultural Statistics, New Delhi.
- 10. Thmpson, S. K. (2012). Samplig, 3rdEdn., Wiley
- 11. ParimalMukhopadhyay: New Central Book Agency: Sampling Theory

## Semester II

## ST15: Practical I

- 1. Introduction to Statistical Software:
  - Classification, tabulation and frequency tables (univariate and bivariate data).
  - Graphical representation of data.
  - Summary statistics.
  - Scatter diagram, correlation, Regression.
- Matrices: Properties of Matrices (rank, inverse, transpose, determinant etc), Getting vectors in row/column space and null space of the given matrix. (1)
- Definiteness of Matrix, eigen values and eigen vectors of a matrix, algebraic and geometric multiplicity of an eigen value, etc. Computing power of a given matrix using spectral decomposition. (1)
- Inverse of a square matrix (by direct method and partitioning method), g-inverse, MP g-inverse.
  (1)
- Gram-Schmidt orthonormalization: Forming an orthogonal matrix of specified order using Gram-Schmidt orthogonalization. (1)
- 6. Solution of System of Linear Equations using Gauss elimination, Gauss Jorden, Gauss Seidal and Gauss Jacobbi methods.
  (2)
- Classification and Reduction of Quadratic forms, Verification of Cayley-Hamilton theorem (1)
- Model sampling from discrete, continuous and mixture distribution (Use inversion method if necessary). (1)

9. Model sampling from bivariate probability distribution. Computation of probability distribution	ability of				
10. Computation of probability of non-central $\chi^2$ , t, F-distributions.	(1)				
11. H-T estimator and PPS, $\pi$ PS (Midzuno) designs. Confidences Interval of estimator					
(2)					
12. Stratified Random Sampling					
• Various kinds of allocation and estimation of population total and mean with S.E.					
• Post stratification.	(1)				
13. Stratified Random Sampling:					
Ratio method of estimation					
Regression method of estimation	(1)				
14. Circular Systematic Sampling					
15. Cluster Sampling with equal and unequal cluster size					
16. Jackknife and bootstrap methods of estimation( for Ratio, Regression coefficient,					
Coefficient of variation, Correlation coefficient)	(1)				
17. Two stage sampling	(1)				
18 Numerical methods: (i) Solution to Simultaneous Bivariate equations by Newton Raphson					
method (ii) Newton's Interrpolation for bivariate functions (iii) Evalution of double integral					
by Trapezoidal rule and Simpson"s rule	(1)				

## **Semester II**

## **ST 21: Probability Theory**

#### Unit 1

Review of algebra of sets, sequence of sets, limsup, liminf and limit of asequence of sets, Classes of sets, field, sigma field, minimal sigma field, Borel fields, measurable space, monotone classes, Measurable function, Real and Vector valued random variables, simple random variable, random variable as a limit of sequence of simple random variables, Probability measure on a measurable space, probability space, properties of probability measure: continuity, mixture of probability measures, Lebesgue and Lebesgue-Steltjes measures. (15L)

#### Unit 2

Distribution function, decomposition of a distribution function, discrete and continuous type random variable, Correspondence theorem, Expectation of simple random variable, non-negative random variable, arbitrary random variable, properties of expectation, moments, moment inequalities. (15L)

#### Unit 3

Convergence of a sequence of random variables, convergence in probability, convergence in distribution, convergence in rth mean, almost sure convergence, their inter-relations, Slutkey's Theorem, convergence theorem for expectations,(15L)

#### Unit 4

Independence of events, class of independent events, independence of classes, independence of random variables, expectation of the product of independent random variables, equivalent definitions of independence, Kolmogorov 0-1 Law, Borel 0-1 criterion, Khintchin's WLLN, Strong Law of Large Numbers (SLLN) (Statement only), Central Limit Theorem (CLT), Levy continuity theorem, CLT for i.i.d. random variables, Liaponove's form, Lindeberg Feller form and their applications. (15L)

#### **Books Recommended**

1. Athreya, K. B. and Lahiri S. (2006). Probability Theory, Hindustan BookAgency,

2. Bhat, B. R. (2007). Modern Probability Theory: An Introductory Text Book, New Age International

3. Billingsley, P. (1995). Probability and Measure, 3rdEdition, John Wiley, NewYork

4. Chung, K. L. (2001). A Course in Probability Theory, Third Edition, Academic Press, London

5. Gut, Allan (2005), Probability: A Graduate Course. Springer, New

# Semester II

## **ST 22 : Regression Analysis**

### Unit 1

- (i) Brief review of simple linear regression: assumptions, least square (LS) estimators of parameters, standard error of estimators, testing of hypothesis for coefficient of regression
- (ii) Multiple regression: Standard Gauss-Markov (GM) setup, least square (LS) estimation with and without restrictions on parameters, variance and covariance of LS estimators, GM theorem (statement and proof for  $Var(\varepsilon) = \sigma^2 I$  and  $Var(\varepsilon) = \sigma^2 V$ ), estimation of error variance (with and without correlated observations) [15L]

#### Unit 2

- (i) Confidence intervals and regions, testing of hypothesis for one and more than one linear parametric functions, testing of hypotheses about parallelism (slopes), equality of intercepts, congruence of two simple regression models, lack of fit test
- (ii) Polynomial regression model (one and two regressors), orthogonal polynomial regression, cubic spline regression model
  [15L]

#### Unit 3

(i) Diagnostic checks and correction: graphical techniques, tests for normality (Shapiro test, Anderson-

Darling test), uncorrelatedness, homoscadasticity; Criteria for model adequacy:  $R^2$ , adjusted  $R^2$ ,

Mallow's Cp etc.

- (ii) Durbin Watson test, estimation of parameters in autocorrelation
- (iii) Outlier, leverage points, influential points, PRESS statistic, Cook's D statistic
- ( iv)Multicollinearity : consequences, tools for detection and remedies,Ridge Regression. [15L]

## Unit 4

- (i) Non-linear regression: linearizationtransforms, their uses and limitations .Box and Cox transformations
- (ii) Generalized linear model:introduction to link functions such as binomial, inverse binomial, inverse Gaussian and Gamma.
- (iii) Logistic regression: Logit transform, ML estimation, tests of hypothesis, Wald test, LR test,

score test, test for overall regression

(iv) Poisson regression: log link transform, ML estimation, tests of hypothesis, Wald test, LR test,

score test, test for overall regression.

[15 L]

## **Books Recommended**

1. Cameron, A. C. and P. K. Trivedi (1998). Regression Analysis of Count Data, Cambridge

- 2. Draper, N. R. and Smith, H. (1998). Applied Regression Analysis, John Wiley, ThirdEdition.
- 3. Hosmer, D. W. and Lemeshow, S. (1989). Applied Logistic Regression, Wiley.
- 4. Kleinbaum, D. G. & Klein, M. (2002). Logistic Regression: A Self-Learning Text, Springer
- 5. McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, Chapman& Hall.
- 6. Montgomery, D. C., Peck, E. A. and Vining, G. G. (2003). Introduction to LinearRegression Analysis, Wiley.
- 7. Neter, J., W., and Kutner, M. H. (1985). Applied Linear Statistical Models, Wiley.
- 8. Ratkowsky, D. A. (1983). Nonlinear Regression Modelling, Marcel Dekker, London.
- 9. Ruppert, D., Wand, M. P. and Carroll, R. J. (2003) Semiparametric Regression, Cambridge University Press.
- 10. Seber, G. E. F. and Wild, C. J. (1989). Nonlinear Regression, Wiley.
- 11. Weisberg, S. (2005). Applied Liner Regression, Wiley.

# Semester II

# ST 23 Statistical Inference I

## Unit1

Fisher information and information matrix, concept of Sufficiency, Neyman factorization theorem, likelihood equivalence, minimal sufficiency, construction of minimal sufficient statistics, special classes of distributions:one parameter exponential family, multiparameter exponential family, Pitman family, minimal sufficient statistic for special classes of distributions.

(15L)

## Unit 2

Completeness, bounded completeness, complete sufficient statistics, special classes of distributions admitting complete sufficient statistics, ancillary statistic, Basu's theorem and its applications, estimability of parametric function, Cramer-Rao inequality, minimum variance unbiased estimators (MVUE), necessary and sufficient conditions for existence of MVUE, Minimum variance bound unbiased estimators (MVBUE), Chapman-Robin Bounds (without proof), Bhattacharya Bounds (without proof), Rao- Blackwell theorem, Lehman-Scheffe theorem.

(15L)

## Unit3

Problem of testing of hypothesis, simple and composite hypotheses. randomized and nonrandomized tests, most powerful test, Neyman-Pearson Lemma (with proof), power function of a test, existence of UMP tests for one-sided alternatives in one parameter exponential family and Pitman family, UMP tests for two sided alternatives, their existence and non-existence, unbiased test, UMP unbiased test, Monotone likelihood ratio property and its applications. (15L)

### Unit4

Concept of confidence intervals, relation with testing of hypothesis, shortest expected length confidence intervals, uniformly most accurate confidence bounds.

Introduction to Bayesian estimation, prior and posterior distributions, loss functions, principle of minimum expected posterior loss, quadratic and other common loss functions, conjugate family of prior distributions and its examples. (15L)

## **Books Recommended**

1. Casella, G. and Berger, R. L. (2002). Statistical Inference. Duxbury AdvancedSeries, Second Edition.

2. Efron, B. and Hastie, T. (2016). Computer Age Statistical Inference: Algorithms, Evidence and Data Science. Cambridge University Press

3. Kale, B.K. & Muralidharan, K. (2015) Parametric Inference: An Introduction, Alpha Science International Ltd.

4. Lehmann, E.L. and Casella, G. (1998). Theory of Point Estimation. Springer, NewYork

5. Lehmann, E. L. and Romano, J. (2005). Testing Statistical Hypotheses, Springer

6. Rao, C. R. (1995). Linear Statistical Inference and its Applications, Wiley

7. Rohatgi, V. K. and Saleh, A.K. Md. E. (2001). Introduction to Probability and Statistics, John Wiley & Sons, New York.

8. Shao, J. (2003). Mathematical Statistics, Springer-Verlag, New, New York,

# Semester II

## ST 24: Multivariate Analysis

## Unit 1

Exploratory multivariate Data Analysis: Sample mean vector, Dispersion Matrix, Correlation Matrix, Linear transformation and its mean and variance, graphical interpretation. Principal component Analysis (by using covariance and correlation method, standardized method), Factor analysis (Their models, rotation types), Canonical correlation with real life examples. [15L]

## Unit 2

Cluster analysis (Hierarchical and Non hierarchical, Agglomerative, Single, complete, average, Wald's linkage, K- mean clustering method, qualitative method clustering) Multivariate normal distribution, Singular and nonsingular normal distribution, mean, variance of multivariate normal distribution, independence of variable, M.G.F, Characteristic function, moments, Distribution of linear and quadratic form of normal variables, marginal and conditional distribution, multiple and partial correlation coefficient (3 random variable case)with examples on each of the topic. [15L]

## Unit 3

MLES of parametric of multivariate normal distribution and their sampling distribution, Tests and confidence region for the mean when dispersion matrix is known, Wishart distribution (generalized case of chi-square) and its properties, Hotelling T<sup>2</sup> statistics and uses of its distribution, Beharen- Fishers problem , confidence region for mean vector of multivariate normal distributions. [15L]

## Unit 4

MANOVA technique , Likelihood ratio test , Test for equality of dispersion matrices, Discrimenant analysis ( by using prior probabilities , by using cost ), Fisher Discrimenant analysis, Mahalanobis D<sup>2</sup> Statistics [15L]

## **Books Recommended :**

1. Anderson, T. W. (1984). Introduction to Multivariate Analysis, John Wiley

2. Fang ,K., Kotz, S., Ng K. W. (1990). Symmetric Multivariate and RelatedDistributions, Chapman and Hall

3. Härdle, W. K. & Simar, L. (2012). Applied Multivariate Statistical Analysis, Springer, New York

4. Härdle, W. K., Hlávka, Z. (2007). Multivariate Statistics: Exercises and Solutions, Springer, New York

5. Johnson R.A. & Wichern, D.W. (1988). Applied Multivariate Statistical Analysis, Prentice Hall Inc.

6. Kotz, S., Balakrishnan N. and Johnson N. L. (2000). Continuous MultivariateDistributions,

Volume 1, Models and Applications, John Wiley & Sons,

7. Kshirsagar, A. M. (1983). Multivariate Analysis, Marcel Dekker

8. Mardia, K. V. and Jupp, P. E. (2000), Directional Statistics, John Wiley & Sons

9. Morrison, D.F. (1990). Multivariate Statistical Methods, McGraw Hill Co.

# Semester II

# ST 25 Practical II

- 1. Exploratory Multivariate data Analysis. (Sample mean, variance and covariance matrix, Correlation Matrix )
- 2. Principal component Analysis (covariance & Correlation technique and their interpretation )
- 3. Factor analysis ( PCA., MLE, all Rotations and their interpretation )
- 4. Cluster analysis (Single, Complete, Average, Wards, k-mean linkage method)
- 5. Canonical correlation
- 6. Multivariate Analysis. (Multivariate normality, Marginal, Conditional, Q-Q plot, contour plot)
- 7. Model Sampling from Multivariate Distribution, and computation of MLE's of Parameters.
- 8. Application of Hotelling T<sup>2</sup> Statistic
- 9. MANOVA technique
- 10. Likelihood Ratio Test( Equality of means, Equality of variance, R=0)
- 11. Discriminant Analysis (Fishers linear discriminant function )
- 12. Simple and Multiple Regression and Regression diagnosis
- 13. Selection of variables in Multiple regression and lack of fit
- 14. Transformation and weighting to correct model inadequacies.
- 15. Polynomial regression model (one and two regressors)
- 16. Multicolinarity and ridge regression.
- 17. Spline Regression
- 18. Logistic Regression
- 19. Poisson Regression
- 20. Application of Central Limit theorem and Weak law of large number